Bi-phase methodology for sensitivity analysis of complex models, applied to the model of evaluating resilience in transport networks.

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ABSTRACT: Sensitivity analyses identify the importance of the model parameters and variables, providing a profound knowledge of the model. Most of the sensitivity analysis implies a local evaluation of the variability of the parameters. However, in those cases where the model involves a large number of degrees of freedom, these studies become highly time consuming and incapable of obtaining conclusive solutions. This paper presents a bi-phase approach, by integrating a local into a global sensitivity method. This methodology is recommended in those multidimensional models that make other approaches inefficient. The global phase is based on the statistical distribution of the variables, providing a robust and reliable solution, with low computational costs. With the aim of showing the potential of this approach, the method is applied to a complex existing model. This model aims to evaluate the resilience in transport networks when affected by an extreme weather event. As expected, this sensitivity analysis shows the large influence of the hazard intensity, but also shows how other variables, as the size of the area affected by the hazard and the role that the users play, can modify the values of the sensitivity analysis significantly.

KEY WORDS: Sensitivity analysis; Resilience; Hazards; Extreme Weather; Latin Hypercube; On-At-a-Time; Local Sensitivity; Global Sensitivity; Vulnerability.

1 INTRODUCTION

A variety of extreme weather events, including river floods, rain induced landslides, droughts, winter storms, wildfire, and hurricanes, have threatened and damaged many different regions worldwide. These events have a devastating impact on critical infrastructure systems resulting in high social, economic and environmental costs. For this reason, it is imperative to develop a mathematical tool that is able to measure systematically the impacts of extreme weather events on infrastructures.

Once the model for assessing the impacts of climatological hazards is developed, a sensitivity analysis should be carried out. A sensitivity analysis identifies the influence of each parameter on the outputs of the model, permitting a profound knowledge of its behaviour. In addition, the definition of the inputs will be more efficient after studying how these parameters modify and influence the model.

Different methodologies to address a sensitivity analysis have been developed previously. These methods can be differentiated between local methods and global methods, the former focuses on estimating the local impact of a parameter on the model outputs.

Global techniques are based on sampling methods which scan, in a random or systematic way, the complete range of the parameters involved in the model. Selection of the sampling strategy is crucial to the sensitivity analysis.

This paper presents a bi-phase sensitivity analysis, by integrating a local into a global sensitivity method. This methodology is recommended in those multidimensional models that make other approaches inefficient. Especially, in those cases where the model involves numerous degrees of freedom, since other methodologies become highly time consuming and incapable to obtain conclusive solutions.

The paper is organized as follows; Section 2 describes different methodologies to develop a sensitivity analysis. Section 3 introduces the proposed bi-phase sensitivity analysis; in Section 4, the model to evaluate resilience, used to apply the sensitivity analysis, is briefly explained, and in Section 5, a case of study is developed. Finally, in Section 6 some conclusions and future research lines are drawn.

2 SENSITIVITY ANALYSIS

A sensitivity analysis can be defined as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input" [1].

Some of the main reasons to develop a sensitivity analysis are highlighted:

- The model parameters require additional research for strengthening the knowledge base, thereby the output uncertainty is reduced.

- Some model parameters might have a negligible contribution, therefore they can be eliminated from the final model. This would result in a reduction of the required computational time.
- Bigger effort should be made in defining those variables and parameters with larger contribution into the model. The sensitivity analysis allows the identification of those important variables and parameters.
- The consequences in the results can be determined when changing a given input parameter or variable.

Different methodologies to analyse the sensitivity have been developed previously.

An initial classification can differentiate between global and local methodologies, based on two different levels to carry out a sensitivity analysis.

2.1 *Local methodologies*

A local sensitivity analysis evaluates sensitivity at one point in the parameter hyperspace. The local techniques aim to estimate the local impact of a parameter on the model output. A sensitivity coefficient is obtained, which is basically the ratio of the change in output to the change in input while all other parameters remain constant [2].

Some methodologies to develop a local analysis are, (a) differential sensitivity analysis, based on partial differentiation of the model in aggregated form; (b) one-at-a-time measures, which is one of the simplest method to develop a sensitivity analysis, is based on repeatedly varying one parameter at a time while holding the others fixed and (c) the sensitivity index, which calculates the output percentage difference when varying one input parameter from its minimum value to its maximum value.

2.2 *Global methodologies*

Global sampling methods scan in a random or systematic way the entire range of possible parameter values and possible parameter sets. These techniques analyze the whole parameter space at once. Some methodologies to develop a global analysis are, (a) simple random sampling, using Monte-Carlo analysis. This method works by generating a random value of the variable analysed and scaling this one to the target variable via its probability distribution. (b) Stratified sampling, which represents an improvement over simple random sampling by forcing the sample to conform to the whole distribution being analysed.

Any reduction in the number of simulations required for a Monte-Carlo analysis will result in a reduction in computational effort, for that reason, some techniques have evolved and can outperform the simple random sampling.

As an example, (c) the Latin-Hypercube simulation is a method of sampling that can be used to produce input values for estimation of expectations of functions of output variables [3]. The method works by dividing the input into strata and then generating samples so that the value generated for each parameter comes from a different stratum [4].

3 METHODOLOGY

The applied methodology is an integration of two phases, a local into a global sensitivity method. This paper presents a combination of One-At-a-Time (OAT) for the local sensitivity and Latin Hypercube (LH) sampling for a global approach, [5].

3.1 *Upper level approach*

In order to address the first phase of this sensitivity methodology, a sampling strategy is carried out.

The selected global sampling procedure is the LH that allows the reduction of the sample size. Due to the importance of the pairing procedure, the method Translational Propagation algorithm proposed by [6] has been implemented in this analysis. The main advantage of this methodology is that it requires virtually no computational time. When the sample is obtained, the local sensitivity analysis can be accomplished as follows. Considering that the total space is covered and the sample is a reliable and robust representation of the entire space, the model is evaluated for each point of the sample, using a local sensitivity analysis.

3.2 *Lower level approach*

On the other hand, the second phase of this methodology is based on a local technique to evaluate the sensitivity.

According to the OAT technique, the analysis is performed by modifying every variable in each sample point in a percentage to calculate the corresponding model response in that close point. It is important to modify only one variable each time to identify the behaviour of the modified variable. Measuring the variation, according the OAT methodology, the sensitivity in each point is captured.

This local method is as simple as efficient, however this process can become quite intensive with larger models. Then, instead of applying it in a large number of points to cover the entire range of the parameters, a global methodology has been chosen to obtain a sample of points that represents the different variables.

The formulation to assess the sensitivity is based on the concept of derivative, that is

$$\xi = \frac{R_d^+(x,Y) - R(Z)}{d}, \qquad x, Y \in Z, \quad (1)$$

where Z is the set of variables involved in the model, x, is the modified variable and Y, the subset of variables which remain constant. R is model response calculated for the initial parameter set Z and R_d^+ is the model response when one parameter has been increased by a percentage, d. Sensitivity, denoted by ξ, is a dimensionless parameter.

The percentage, d, is also a critical point, as small values can show the instabilities of the model, being this behavior not according with the tendency of the model. However, if this value is too large, the derivative loses its meaning.

4 MODEL

This methodology to develop a sensitivity analysis, is implemented for a complex model. In this case, the model evaluates the behaviour of a traffic network when an extreme weather event takes place, determining the resilience of the transport network ([7]).

The most common definition of resilience was given by [6], as "the capacity to absorb shocks gracefully", and a further description of this can be found in [10].

With the aim of quantifying the resilience, [7] propose a "Dynamic Equilibrium-Restricted Assignment Model" (DERAM), which allows the simulation of the network behaviour when a disruptive event occurs. This approach permits the inclusion of the stress level of the system together with the extra cost generated by the hazard. This model proposes that the network behaviour is restricted by a system impedance, α.

The perturbation resilience is defined between (0, 100), 100% being the optimum value. Moreover, a cost threshold is included to assume the system break-down. This value restricts the perturbation resilience and is the limit-state associated with the failure of the travel cost network due to the extreme overcost generated by a strong perturbation. Although the system could theoretically recover, it would imply an unacceptable effort by the system.

With the goal of improving the resilience of a network when affected by a hazard and having a better understanding of the effects of extreme weather events, the weaknesses of the system should be identified. Therefore, the resilience of a network can be enhanced with the improvement of the most influential variables of the model. With this aim, a sensitivity analysis is carried out.

5 STUDY CASE

This methodology is applied in a simple traffic network to analyze the sensitivity of a set of variables.

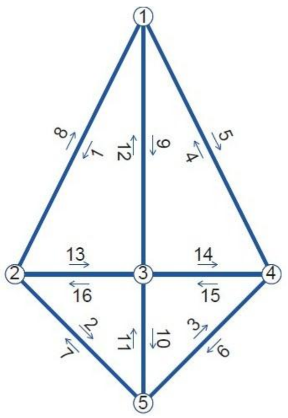

Figure 1. Network.

This traffic network consists of 5 nodes (cities), 16 links (roads) and 10 routes, (see Table 2).

The set of variables used for the sensitivity analysis together with its characteristics are shown in Table 1.

Table 1. Statistical distributions of the parameters

Links	Parameters	Distribution	Distribution parameters
Affected	*h(t)*	Beta	α=1.2,β=3
No-affected	*h(t)*	Deterministic	0.001
Affected	p_a	Beta	α=1.2,β=2.5
No-affected	p_a	Deterministic	0.001
All links	α	Beta	α=1.2,β=1.2
Affected	β_a	Gamma	k=2.9,θ=0.5
No-affected	β_a	Deterministic	0.83
Affected	γ_a	Gamma	k=7,θ=0.37
No-affected	γ_a	Deterministic	4

Where β_a and γ_a are parameters related to the traffic characteristics; *h(t)* is the hazard intensity whose range is (0,1), and p_a is the specific sensitivity of each link to a given hazard. For instance, in the case of pluvial flooding, p_a depends on the catchment area, slope of the road, type of pavement, existence of element of protection, etc. Subscript a implies association with link a.

For the following examples, the variables analysed will be the ones related with the hazard, i.e., *h(t)*, p_a and α. The sample size selected has been 25 points and the percentages of variation, d, are 1, 5, 10%. Figures 3-6 show the results associated with the percentage of 10%, since the results for the other percentages follow a similar tendency, excluding the case of the 1%, where some numerical instabilities of the model were identified.
For a detailed study of the rest of the parameters, see [9].

The sensitivity analysis is carried out by presenting different scenarios, to figure out the influence of the climatological parameters in the system. Moreover, a second goal is to demonstrate that the sensitivity depends on other factors as the area affected by the hazard, or the different possibilities that the users have to avoid the hazard (redundancy of the network).

In order to reach this goal, two sections are presented, (a) depending on the area affected by the hazard and (b) depending on the redundancy of the network.

5.1 Considering different affected areas.

To increase the knowledge and the understanding of the effect of the intensity of the hazard, three cases has been developed, since the influence of this parameter is crucial.

To that end, each of the cases has a different area damaged by the hazard, see Figure 2. First case (green area in Figure 2), only the links between the node 2 and node 3 are exposed to the climatological event. Second case (red area in Figure 2), half of the network is altered by the perturbation and, finally, in the third case, the entire network is affected.

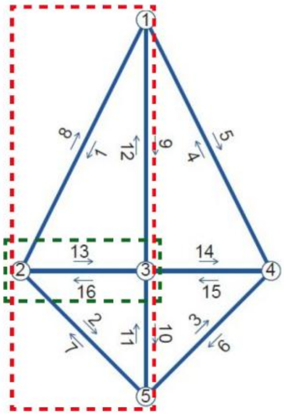

Figure 2. Areas affected by the hazard.

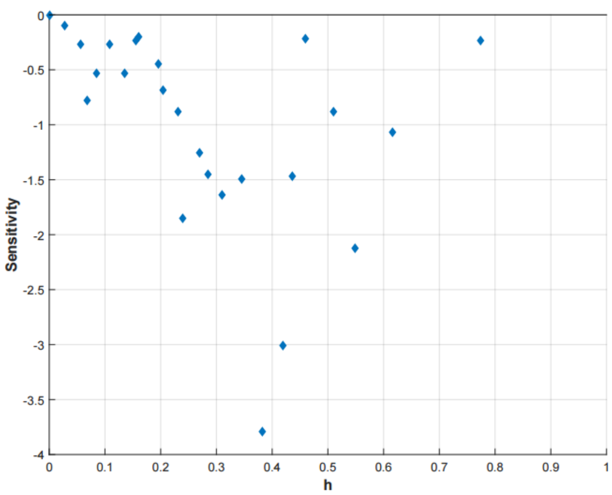

Figure 3. Sensitivity of h, links damaged 13-16.

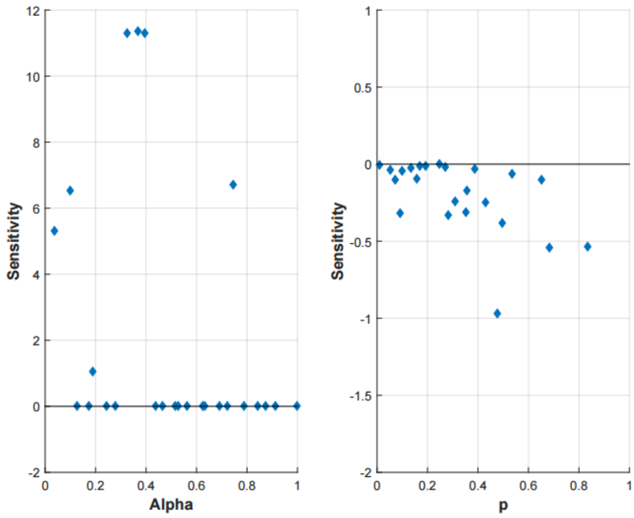

Figure 4. Sensitivity of alpha and p_a, links damaged 13-16.

Analyzing the first case, where only the links 13-16 are affected by the hazard, it is evident that the intensity of the hazard, *h(t)* (Figure 3) has a larger sensitivity than p_a (Figure 4). In Figure 3, most of the points are within the range of 0 to -2. Some of them can reach higher values from -2 to -4. On the other hand, the sensitivity values of p_a do not go over -1, remaining a high proportion below -0.5.

It is noted that an increment in the values of *h(t)* and p_a implies a reduction of the resilience, as evidenced by the negative sensitivity of these parameters.

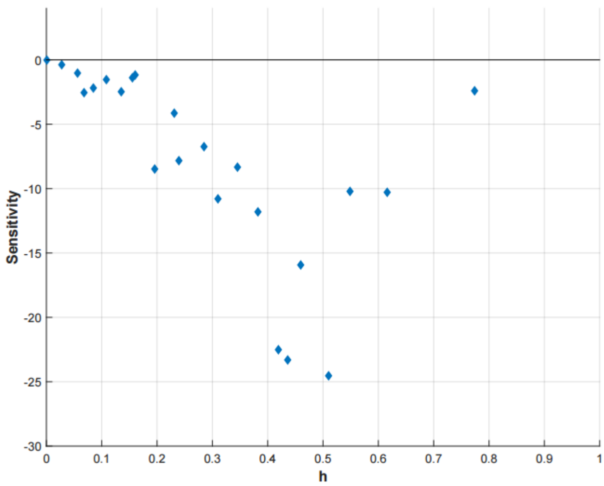

Figure 5. Sensitivity of h, half network damaged.

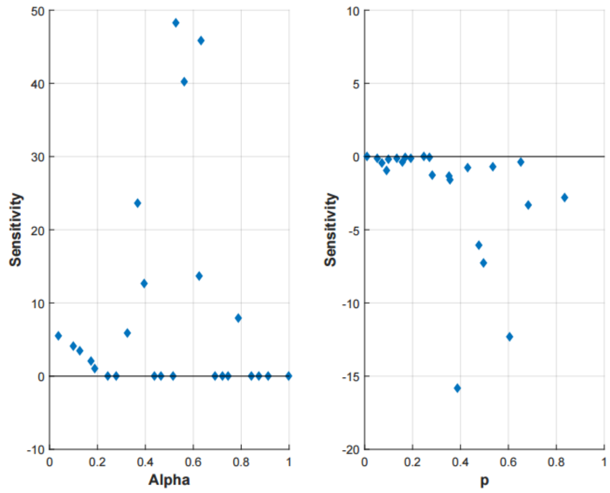

Figure 6. Sensitivity of alpha and p_a, half network damaged.

In the second case, Figures 5 and 6 show the sensitivy of the three parameters, i.e, *h(t)*, α and p_a when half of the network is exposed to the hazard. As can be noted from the Figures, the sensitivity of all the parameters related to the hazard increases substantially. When compared with the previous example, the values are approximately 10 times larger for h(t) and p_a and five times for α. This happens because the area damaged by the hazard is larger for the second case. In addition, the greater sensitivity of *h(t)* is corroborated with this example.

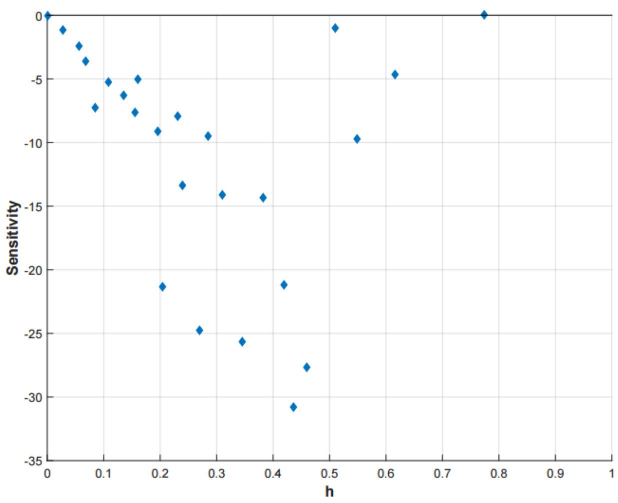

Figure 7. Sensitivity of h, all links damaged.

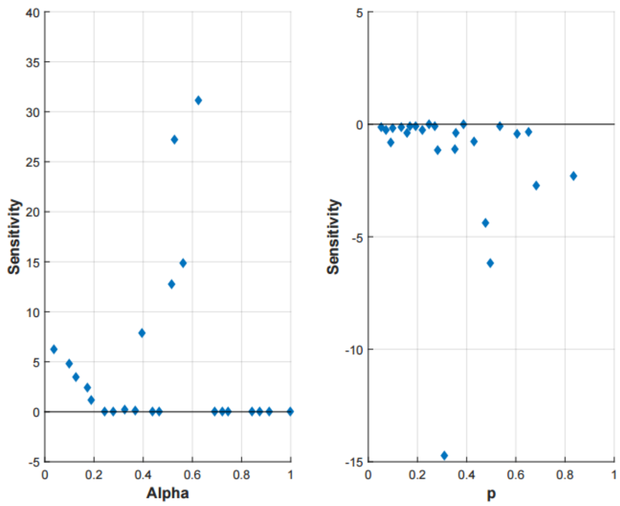

Figure 8. Sensitivity alpha and p_a, all links damaged.

Finally, the third case, shown in Figures 7 and 8, confirms that the values for the sensitivity of *h(t)* are even larger, since that the whole network is damaged by the hazard. Furthermore, the values of p_a continue to be smaller that *h(t).*

5.2 Considering the redundancy.

In the previous cases, the focus was mainly in two parameters, *h(t)* and p. However, the parameter alpha, the system impedance, reaches the highest values, becoming a very relevant variable. It is noted that an increment of α implies an increment of the resilience.

The influence of alpha on the resilience index is larger when users play an active role, that is, when they can improve their situation by changing their routes.

Therefore, the sensitivity of this parameter is going to depend mainly in the options that the users have to change, that is, the redundancy of the system.

Table 2. Routes, defined by the links.

Routes	Example 1	Example 2
1-13-14	✓	✓
9-10-3	✓	✓
2-11-12	✓	✓
13-2	✓	✓
3-4	✓	✓
7-8	✓	✓
5-15-16	✓	✓
5-6-7	✓	✓
9-10	✓	
1-2	✓	

To prove the considerable variability of this parameter, a second example has been introduced. For this new case, the number of routes has been reduced, as shown in table 2, using the case where half of the network is affected. Consequently, the change options of the users have been reduced and the active role that they can play has been decreased.

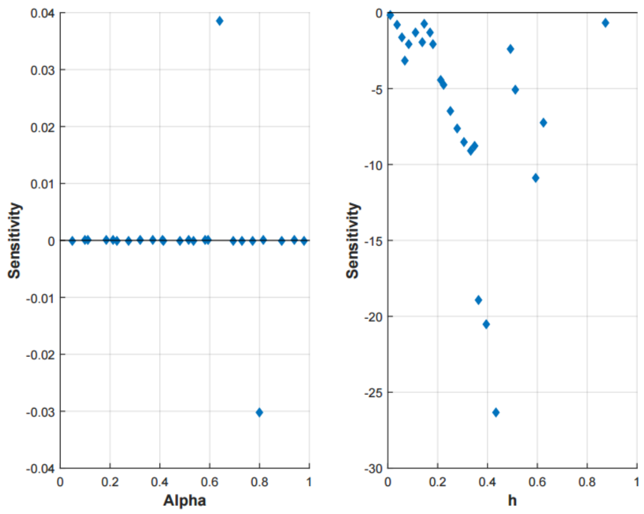

Figure 9. Sensitivity alpha and h, example 2.

In Figure 9 is shown that when the change possibilities are reduced, the sensitivity of alpha decrease largely. In this case, example 2, the sensitivity of alpha is negligible, when in all the previous examples (see Figures 3-8), alpha is the most sensitive parameter.

6 CONCLUSIONS

The selection of an adequate methodology to analyse the sensitivity of the parameters should include a statistical approach to reduce the computational times and the number of chosen points to cover the entire range of the parameters.

For that reason, a mixed methodology to analyse the sensitivity is proposed, which include local (One-At-a-Time) and global techniques (Latin Hypercube). This kind of methodology is justified when a large number of variables are involved, because local methods are very efficient but they do not cover the entire space; whereas, global methods provide a robust and reliable approach but the computational cost could be too high in complex models.

In addition, the following aspects can be highlighted:

- The pairing procedure known as the Translational Propagation algorithm, has been implemented, which requires minimal computational times.
- The sensitivity analysis shows the important role of the hazard intensity.
- The area affected by the hazard modifies the sensitivity of the hazard intensity substantially. Therefore, when the zone exposed to the hazard increases, the values of the sensitivity of this parameter rise.
- The analysis reflects the significant influence of the system impedance, with the option of becoming as important as the hazards intensity. However, this only happens when users play an active role, that is, when they can improve their situation by changing their routes. If the users do not have route options and the redundancy of the network is insignificant, it is demonstrated that the parameter is not relevant in the sensitivity analysis.

Future research will provide an extension of this methodology, including aspects such as the topology of the network, the road capacity and the traffic demand.

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