Dynamic Restricted Equilibrium Model to Determine Statistically the Resilience of a Traffic Network to Extreme Weather Events

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ABSTRACT: Extreme weather events lead transportation systems to critical situations, which imply high social, economical and environmental costs. Developing a tool to quantify the damage suffered by a traffic network and its capacity of response to these phenomena is essential to reduce the damage of this hazard and to improve the system. With this aim, a statistical analysis of the resilience of a traffic network under extreme climatological events is presented. The resilience of a traffic network is determined by means of a dynamic restricted equilibrium model together with a travel cost function that includes the effect of weather on a traffic network. The cost function parameters related to the hazard effect are assumed as random, following Generalized Beta distributions. Then, the fragility curves of the target traffic network are defined using the Monte Carlo method and Latin Hypercube sampling. Fragility curves are a useful tool to analyse of the vulnerability of a traffic network, assisting in the decision-making for the prevention and response to the extreme weather events.

1. INTRODUCTION

The loss of serviceability of a traffic network occurs several times during its service life. Nevertheless, this problem should be limited to a certain level of acceptable risk and should be easily recoverable. Extreme weather events lead the transportation systems to anomalous situations and in some cases, even to critical ones, which imply a high social, economical and environmental cost. The quantification of the damage suffered by a traffic network and its capacity of response to these phenomena is of great importance in order to identify and enhance any network weaknesses, generating more efficient designs.

In recent years a holistic concept has been used to define the capacity of a system potentially exposed to hazards, to adapt by resisting or changing in order to reach and maintain an acceptable level of functioning (UN/ISDR (2004)), i.e. the resilience. Quantifying the resilience of a system is not a straightforward task, because it includes aspects such as robustness, redundancy, resourcefulness, adaptability, ability to recover quickly, among others (Bruneau et al. (2003); Murray-Tuite (2006); Park et al. (2013)). Despite the complexity of this task, some numerical models exist which quantify the resilience of traffic systems (Ip and Wang (2009); Vugrin et al. (2010); Henry and Ramirez-Marquez (2012); Chen and Miller-Hooks (2012); Nogal et al. (2014b)). Nevertheless, the disruptive events considered in most of the existing literature are not related to climatological events but modification of the traffic network characteristics. With this purpose, some authors such as Lam et al.

(2008) or Nogal et al. (2014a), propose some travel cost functions to capture the effects of the weather on the traffic network.

On the other hand, systems have some capacity of adaptation to a certain degree of perturbation, however, when a disruption occurs in a sudden and intense manner, critical states are generated. When the perturbation is produced by climatological phenomena, the extreme weather events will cause these critical states. According to the latest Intergovernmental Panel On Climate Change report (IPCC (2014)), confidence has increased that some extremes will become more frequent, more widespread and/or more intense during the 21st century. In this present-day context, tools and guidelines to assess the resilience of transportation systems under extreme weather events becomes necessary.

Due to the uncertainties inherent to this problem, a statistical approach guarantees a more realistic and valuable estimation of the response of the systems. Therefore, the aim of this paper is to present a statistical analysis of the resilience of a traffic network under extreme climatological events. The resilience of a traffic network is determined by means of a dynamic restricted equilibrium model together with a travel cost function that includes the weather effect on a traffic network. As a result of the uncertainties of the cost function parameters related to the hazard effect, their values are assumed as random. The Latin Hypercube is used to sample different network models to be analysed under real and synthetic climatological extreme events. Finally, by means of the Monte Carlo Method the fragility curves of the traffic system are obtained.

This is the first time fragility curves have been used to analyse the resilience of a traffic system under extreme weather events, since this statistical tool has been traditionally used in the field of structures (Mander and Basöz (1999); Shinozuka et al. (2000); Ghosh and Padgett (2010)). It is noted that the response of a traffic system is highly non-linear and its analysis becomes tough, especially when the extreme weather is the source of the traffic network disruption. For that reason, the fragility curves become a useful tool to assist in the decision-making for the prevention and response to the hazards.

The paper is organized as follows; Section 2 explains the main parameters required to estimate the resilience of the system by means of the fragility curves. The process to obtain these parameters are described in detail in the next sections. Section 3 deals with the concept of resilience as well as the dynamic model required to assess the resilience index. Section 4 explains how to determine the hazard intensity and Section 5 gives an example to illustrate the performance of the proposed method. Finally, in Section 6 some conclusions and future research paths are drawn.

2. FRAGILITY CURVES

In the context of transportation systems, the fragility curves are a representation of the probability that a specific traffic network exceeds a given damage state, as a function of the hazard degree. The fragility function can be expressed as follows:

$$F_{DS_i}(HD) = P[DS > DS_i | HD), \qquad (1)$$

where HD is a parameter indicating the hazard degree and DS, a parameter indicating the damage state.

Therefore, to obtain these curves, the following variables have to been defined, (a) the hazard degree; (b) the discrete Damage States DS_i; and (c) a variable that allows the quantification of the response of the traffic network and, at the same time that can be related to the damage state, i.e. the resilience. Consequently, the discrete damage states are associated with different resilience levels.

Moreover, the parameter related to the hazard degree has to integrate the main aspects affecting the system resilience and, should be easily computed. As indicated, the response of a traffic system mainly depends on the rapidity with a hazard occurs and its intensity. Considering these two aspects, the slope of the cumulative curve of the hazard intensity function has been successfully used to evaluate HD.

By means of the Monte Carlo Method (MCM), the cumulative distribution functions (CDF) of the resilience associated with different values of HD are obtained. Finally, the fragility curves are defined using the CDF for the resilience levels related to the DS_i.

In the following sections, obtaining the resilience and the hazard intensity functions are explained in depth.

3. RESILIENCE. A DYNAMIC RESTRICTED EQUILIBRIUM MODEL

To estimate the response of a traffic network under extreme weather events it is necessary to consider a parameter that represents the global system response. With this aim the resilience has been selected, which can be defined as the capacity of a transportation network (a) to absorb disruptive events, maintaining its level of service, and (b) to return to a level of service equal to or greater than the pre-disruption level of service within a reasonable time frame. In this paper only the resilience in the perturbation stage is analysed.

The assessment of the traffic network resilience requires a dynamic approach. With this aim, Nogal et al. (2014b) propose a "Dynamic Restricted Equilibrium Assignment Model", which allows the simulation of the network behaviour when a disruptive event occurs.

Considering a connected traffic network defined by a set of nodes and a set of links, for certain origin-destination (OD) pairs of nodes, there are given positive demands which give rise to a link flow pattern when distributed through the network. Then, a dynamic "equilibrium-restricted" state can be obtained when, for each OD pair, the actual route travel cost experienced by travelers entering during the same time interval *tends to* be equal and minimal. Nevertheless the system could be unable to reach this state in such a time interval. The reason that the system does not reach an optimal (or minimum cost) state in a given time interval is because the traffic network behaviour is restricted by a system impedance to alter its previous state. This impedance is due to the actual capacity of adaptation to the changes, the lack of knowledge of the new situation and the lack of knowledge of the behaviour of other users.

It is noted that when a disturbance takes place, the travel costs increase. Nevertheless, these costs can remain in low values if the users modify their route choices, increasing the stress level of the system. As this mechanism of response cost-stress is limited, the larger the disruption, the lower the remaining response capacity. Therefore, the traffic network behaviour when suffering a disruption can be assessed by means of its exhaustion level (portion of used resources), and its evolution with the time by the exhaustion curve.

The proposed approach permits the inclusion of both, the stress level of the system and the extra cost generated by the hazard, when assessing the exhaustion level of the network.

Following the concept of resilience as the capacity of the network to absorb a shock, the perturbation resilience evaluates how far the system is from complete exhaustion. For that reason, Nogal et al. (2014b) evaluate the perturbation resilience, χ_κ, as the normalized area over the exhaustion curve, as indicated:

$$\chi_\kappa = \frac{\int_{t_{p0}}^{t_{p1}} (1-\psi_\kappa(t))\, dt}{t_{p1}-t_{p0}} \, 100, \tag{2}$$

where t_{p0} and t_{p1} denote the initial and the final time of the disruptive event, respectively; and ψ_κ, the exhaustion level associated with a given state of perturbation κ. The perturbation resilience is defined between $[0,100]$. Moreover, a cost threshold is included to assume the system break-down. This value restricts the perturbation resilience and allows the comparison of different resilience indices.

Furthermore, a travel cost function that includes the climatological events has to be considered. More precisely, Nogal et al. (2014a) propose the following expression

$$\tau_a(t) = \tau_{0a} \left\{ 1 + m_a \exp\left[- \left(\frac{\frac{\beta_a X_a(t)}{X_a^{max}} + p_a h(t)}{1-h(t)} \right)^{-\gamma_a} \right] \right\}, \tag{3}$$

where τ_a and τ_{0a} are the actual travel time and the free travel time, respectively; m_a, β_a and γ_a are shape parameters; $X_a(t)$ and X_a^{max} are the actual flow and the link capacity to provide a certain service level; $h(t)$ is the hazard intensity whose range is $[0,1)$, and p_a is the specific sensitivity of each link to this type of hazard. For instance, in the case of pluvial flooding, p_a depends on the catchment area, slope of the road, type of pavement, existence of element of protection, etc. Subindex a implies association with link a.

It is noted that when any hazard does not exist, $h(t) = 0$, Eq. 3 becomes

$$\tau_a(t) = \tau_{0a}\left\{1 + m_a \exp\left[-\left(\frac{\beta_a X_a(t)}{X_a^{max}}\right)^{-\gamma}\right]\right\}, \tag{4}$$

with only three parameters related to traffic features.

We indicate that the knowledge of the OD demands of a given network and their associated route flows allows the calibration of the model, i.e. the definition of these three parameters for the steady state.

The parameter p_a, which includes the local vulnerability of the network, can be defined based on the previous experiences and expert opinion. Due to the uncertainty involved, the proposed approach assumes that p_a is a random variable. More precisely, they are assumed to follow a Generalized Beta Distribution $p_a \sim GBeta(\alpha_a, \theta_a; p_{0a}, p_{fa})$, where α_a and θ_a are the shape parameters, and p_{0a} and p_{fa} are the range of the parameter p_a. This distribution allows the definition of the random variable on the interval $[p_{0a}, p_{fa}]$ (Castillo et al. (2012)). After that, by means of the Latin Hypercube Sampling (LHS), different networks are modelled and analysed under real and synthetic climatological extreme events.

4. EXTREME WEATHER EVENTS

The definition of extreme event depends on the response capacity of the system to this event. For instance, regarding the rainfall, one of the criteria for defining “extreme weather” is associated with the rainfall level which causes floods; and this level depends on the drainage systems, among others. It is highlighted that this extreme rainfall is not necessarily related to its return period. It has been demonstrated that the diary precipitation with the largest return period does not always produce the worst flood, as constant, mild rainfall can be even more damaging.

In this section it is explained how to determine the hazard intensity for the case of the pluvial flood due to long periods of rain as well as intense showers. With this aim, the data of the daily precipitation of Valencia (Spain) has been selected because of the annual weather phenomenon called “cold drop”[1], which causes important floods.

The daily precipitation provided by the European Climate Assessment Dataset (ECA (2014)) has been used to compute the cumulative precipitation and the cumulative drainage in Valencia from 1961 to 2010. The cumulative drainage has been obtained by assuming an average drainage capacity, 450 mm/day, and 148.5 mm/day when the soil becomes saturated. When the capacity of drainage is exceeded by the rainfall amount, the flood occurs. For example, Figure 5, which illustrates the daily precipitation, cumulative precipitation and cumulative drainage in Valencia from 2007 to 2010, shows four floods in this period of time, two in 2007, one in 2008 and other in 2009. It is noted that the drainage capacity has been estimated by comparing the time and duration of the obtained hazards with the real historical events registered.

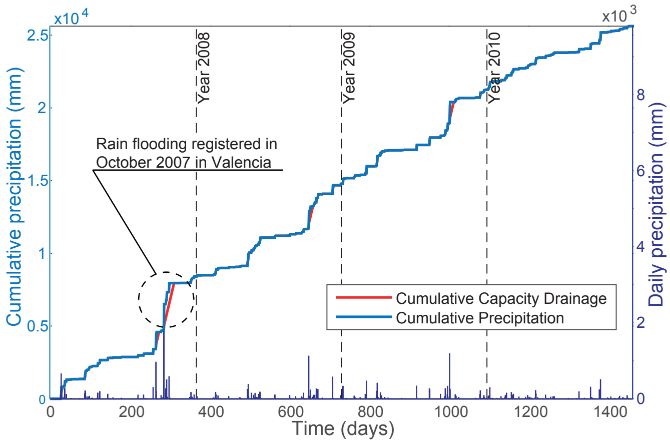

Figure 1: Daily precipitation, cumulative precipitation and cumulative drainage in Valencia.

In order to correctly define the fragility curves, it is necessary to have a great amount of data of hazards, nevertheless, due to the extreme nature of

[1]This phenomenon is associated with extremely violent downpours and storms, but not always accompanied by significant rainfall. This phenomenon usually lasts a very short time, from a few hours to a maximum of four days.

these events, this requirement is fulfilled with difficulty. To overcome this obstacle, synthetic hazards have been proposed. This synthetic response has been generated using real data, and taking into account aspects as the intensity of the hazard, duration, and the rapidity at which the peaks occur.

Figure 2 shows in continuous red colour the 77 pluvial flooding cases observed since 1961 to 2010; and the dashed blue curves represent the 100 synthetic curves considered in this paper.

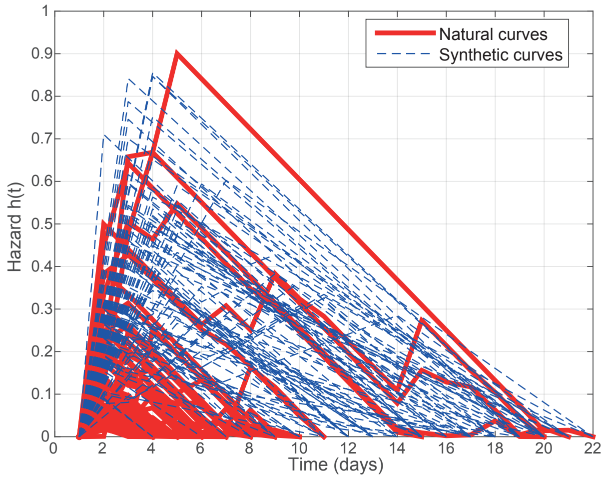

Figure 2: Real and synthetic curves of the pluvial floods obtained from Valencia historical data.

5. ILLUSTRATIVE EXAMPLE

To facilitate the understanding of the proposed method, we consider an illustrative traffic network which consists of 5 nodes and 16 links as indicated in Figure 3. Within the set of possible combinations, only 8 routes and 5 OD pairs have been considered in this simple example.

Table 1 shows the average OD demand for different OD pairs, the routes defined by their links and the route flow associated with the equilibrium state (without any hazard).

Initially the system is in equilibrium, i.e. all users have selected those routes that actually minimize their route travel time. The route flow associated with this equilibrium state is given in Table 1. When the hazard occurs, the system tries to reach a new equilibrium state. However, due to the system impedance this equilibrium is not reached immediately.

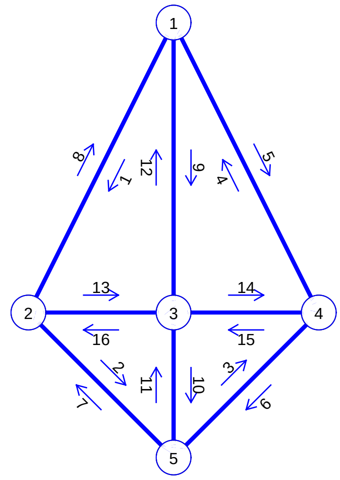

Figure 3: Illustrative network showing nodes and links.

Table 1: OD demand, routes and equilibrium route flow.

OD	Average Demand	Route	Links	Equilibrium Route Flow
1 - 4	100	1	1 13 14	55.416
		2	9 10 3	44.584
2 - 1	50	3	2 11 12	50.000
2 - 5	50	4	13 2	50.000
5 - 1	100	5	3 4 0	50.464
		6	7 8 0	49.536
1 - 2	100	7	5 15 16	45.727
		8	5 6 7	54.273

The parameters of the cost function given by the equation 3 are indicated in Table 2.

With these values, and using the MCM, different traffic networks have been exposed to the real and the synthetic hazards shown in Figure 2.

In each simulation, the link travel cost functions are computed using the link parameters obtained by LHS. Figure 4 shows the link travel cost functions of a selected set of links for different levels of hazard. The lower green line represents the link travel time associated with the state without hazard[2], the upper blue curve provides the link travel time function associated with the maximum level of hazard. The red dashed line marks the $X_{max,a}$ value and the grey band indicates the actual range of link flow

[2]This curve roughly matches with the well known BRP function for ratios $\frac{x_a}{X_{max,a}} < 1.2$)

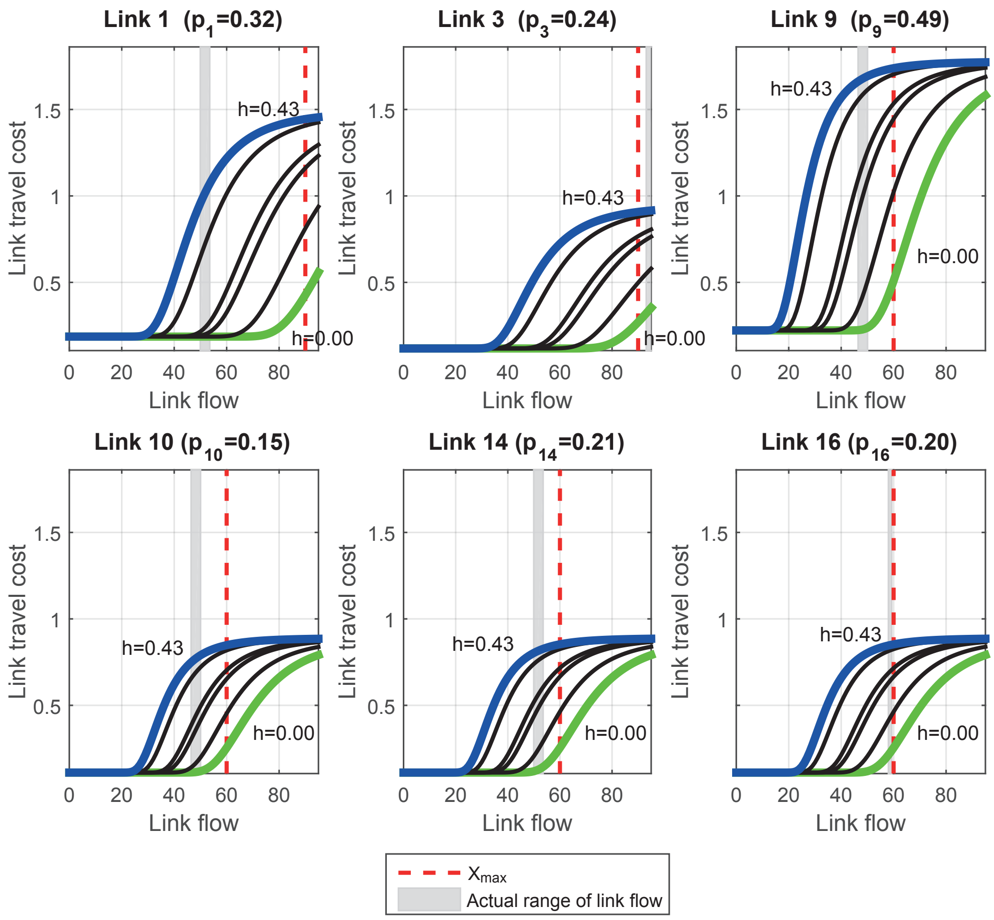

Figure 4: Link travel cost functions for different levels of hazard.

during the process stability-hazard-stability. For instance, it can be appreciated that in link 1, whose parameter $p = 0.32$ in this simulation, the flow rate varies from 50.056 users to 46.44 users as a consequence of the hazard. The link travel cost before the hazard is 0.12 hours and when the hazard level is $h = 0.43$, the link travel cost becomes 0.74 hours.

Subsequently, the resilience associated with each hazard is assessed using Eq. (2) by means of the stress level and the cost function, as shown Figure 5.

After applying the MCM, the CDF of the resilience index for different hazard degrees are computed, as shown by Figure 6.

For this traffic network, four discrete damage states have been considered, viz., negligible, light, light to moderate and moderate, associated with the resilience levels 95%, 85%, 75% and 65%, respectively. Finally, making use of these resilience levels, the fragility curves of the traffic network are obtained, as Figure 7 illustrates. Based on the analysis of the fragility curves it can be stated, for instance,

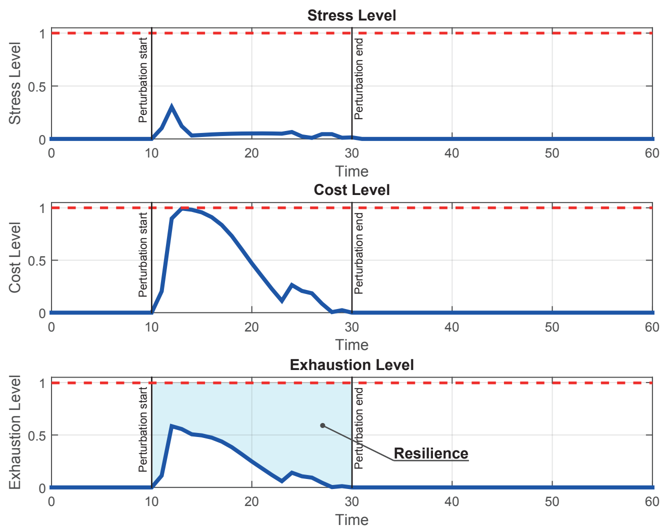

Figure 5: Example of resilience assessment.

Table 2: Parameters of the cost function. $m_a = 7$; $\beta_a = 0.9$ and $\gamma = 5.5$.

Link	Free flow speed (km/h)	Capacity (users)	p Generalized Beta			
			α_a	θ_a	p_{0a}	p_{0f}
1	120	90	3	2	0.2	0.6
2	120	90	10	10	0.15	0.25
3	120	90	2	3.5	0.1	0.4
4	120	90	3	2	0.2	0.6
5	120	90	3	2	0.2	0.6
6	120	90	2	3.5	0.1	0.4
7	120	90	10	10	0.15	0.25
8	120	90	3	2	0.2	0.6
9	90	60	3	2	0.2	0.6
10	90	60	2	3.5	0.1	0.4
11	90	60	10	10	0.15	0.25
12	90	60	3	2	0.2	0.6
13	90	90	10	10	0.15	0.25
14	90	60	2	3.5	0.1	0.4
15	90	60	2	3.5	0.1	0.4
16	90	60	10	10	0.15	0.25

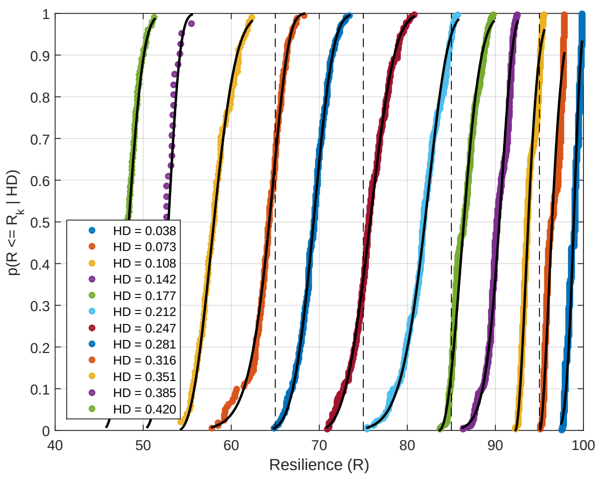

Figure 6: CDF of the resilience for different hazard degrees.

that a hazard of $HD = 0.22$, there is a 98% probability that the damage is worse than light, and 8% probability that the damage is worse than light-to-moderate.

6. CONCLUSIONS

In this paper the fragility curves have been used to determine the traffic network response to a extreme weather event. This representation of the probability that a specific traffic network exceeds a given

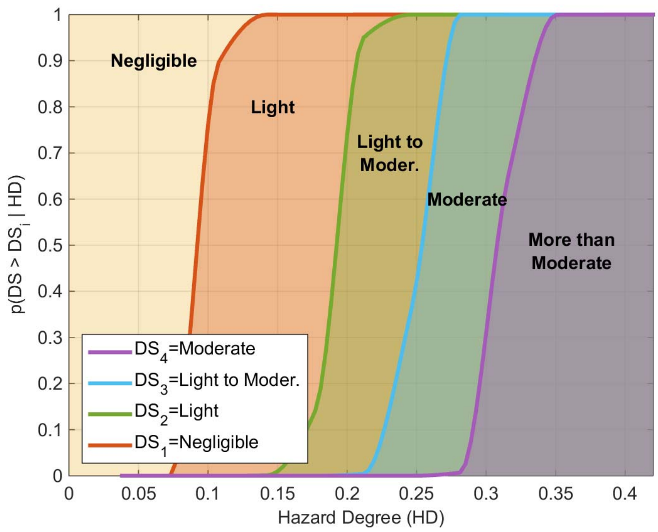

Figure 7: Fragility curves of the traffic network.

damage state, is given as a function of the hazard degree. The resilience has been used to define the damage states and the hazard degree has been computed as the normalized slope of the cumulative curve of the hazard intensity function.

Additionally, the following conclusions can be drawn from this paper:

1. A Dynamic Restricted Equilibrium Assignment Model has been used to compute the resilience of the system. This model takes into account important and complex features of the traffic network such as the stress level, the vulnerability and the capacity of adaptation. Moreover, this model allows a dynamic analysis, which is a key aspect when considering climatological events.
2. By means of a travel cost function which captures the consequences of the extreme weather on the traffic network, time-varying hazards have been introduced coupled with the effect of these hazards on each specific link.
3. The cost function parameter p_a, which includes the local vulnerability of the network, is assumed as a random variable following a Generalized Beta Distribution. This assumption permits the definition of the random variable on the interval $[p_{0a}, p_{fa}]$.
4. The joined probability of the involved parameters is computed by using the Monte Carlo Method together with the Latin Hypercube

Sampling.

5. The uncertainties of the involved parameters make the deterministic approaches inadequate to evaluate the response of a traffic system to this type of disruption. Therefore the probabilistic methods are required to provide a more realistic analysis. More precisely, the fragility curves are a useful tool to evaluate the vulnerability of a traffic network, assisting in the decision-making for the prevention and response to the hazards.

Finally, in following publications, this methodology shall be applied to real networks. However, due to the social sensitivity of the results on real networks, this task has to be addressed carefully. On the other hand, the analysis of fragility curves considering recovery resilience has yet to be carried out. This study will imply a new definition of the *HD* parameter, since the recovery of the traffic network depends on different aspects.

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